SICE Annual Conference 2011
September 13-18, 2011, Waseda University, Tokyo, Japan

Constraint-combined Force/Position Hybrid Control Method with Lyapunov Stability

Fujia Yu[1], Mamoru Minami[1], Tomohide Maeba[1] and Akira Yanou[1]

[1] Okayama University, Tsushimanaka3-1-1 Okayama JAPAN
({ yufujia, minami, maeba, yanou} @suri.sys.okayama-u.ac.jp.)

Abstract: Based on the analysis of the interaction between a manipulator's hand and a working object, a model representing the constrained dynamics of the robot is first discussed. In this paper, we design a new compensation of the system based on the constrained system and prove its convergence in a new way with Lyapunov method, in this way we focus on the motion of the manipulator because the output force is a function of the input torque directly which is not affected by time.

Keywords: Constraint-combined, Grinding robot, Lyapunov Stability.

1. INTRODUCTION

Many researches have discussed on the constraint-combined force/position hybrid control method, to ensure the stabilities of the constrained motion, those force and position control methods have utilized Lyapunov's stability analysis under the inverse dynamic compensation [6]-[8]. But most of these proofs are trying to verify their methods in two different parts [2], [3]: $\lim_{t\to\infty} \boldsymbol{F} = \boldsymbol{F}_d$ and $\lim_{t\to\infty} \boldsymbol{r} = \boldsymbol{r}_d$ here $\boldsymbol{F}$ and $\boldsymbol{F}_d$ are the actual output force and the desired output force, while $\boldsymbol{r}$ and $\boldsymbol{r}_d$ are the actual shape of the manipulator and the desired shape of the manipulator respectively. But in fact because the output force of the end effector can be calculated by the parameters of the manipulator and the input torques directly, this relationship is not affected by time, we can make $\boldsymbol{f} = \boldsymbol{f}_d$ any time, so we do not think it is a good idea to ensure the convergence of the system through the time function of the force, which can only guarantee that the force can get to the desired one at last, especially in the case that we have to control the force all the time, such as the grinding robot. In this paper we will introduce our compensation algorithm and prove its convergence in a new way by Lyapunov method.

Those former classical robot controlling approaches can be classified into two broad categories[8]: impedance control and hybrid (force/position) control. In impedance control, a prescribed dynamic relation is sought to be maintained between the robot end-effector's force exerting to a object constraining the end-effector and position displacement toward the direction vertical to the object's surface [12]. In hybrid control, the end-effector's force is explicitly controlled in selected directions and the end-effector's position is controlled in the remaining (complementary) directions [1].

The hybrid control approaches can be further classified into three main categories: 1) explicit (model based) hybrid control of rigid robots in elastic contact with a compliant environment, e.g., [13]-[14], in which the end-effector force is controlled by directly commanding the joint torques of the robot based on the sensed force error; 2) implicit (position/velocity based) hybrid control of rigid robots in elastic contact with a compliant environment, e.g., [15], in which the end-effector force is controlled indirectly by modifying the reference trajectory given into an inner loop joint position/velocity controller based on the sensed force error; and 3) explicit (model based) hybrid control of rigid robots in hard contact with a rigid environment, e.g., [1],[3].

According to these classified categories, our force/position control approach named as Constraint-Combined Control, which will be introduced in detailed later should be classified into category 3). In all the former force/position controlling methods of hybrid control of category 1) and 2), the contact surface's compliant characteristics must be properly taken into account since it will affect force control procedure. As a result, when contact constraint force is analyzed, process the end-effector contacting with constraint surface is being expressed as a motion equation with spring model, which is a differential function with time-varying.

Eq.(1), which has been pointed out by Hemami [16] in the analysis of biped walking robot, denotes also algebraic relation between the input torque $\boldsymbol{\tau}$ of the robot and exerting force to the working object F_n, when robot's end-effecter being in touch with a surface in 3-D space:

$$F_n = a(\boldsymbol{x}_1, \boldsymbol{x}_2) - \boldsymbol{A}(\boldsymbol{x}_1)\boldsymbol{\tau}, \tag{1}$$

where, $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are state variables. $a(\boldsymbol{x}_1, \boldsymbol{x}_2)$ and $\boldsymbol{A}(\boldsymbol{x}_1)$ are scalar function and vector one defined in following section.

In this paper, position and force control performances of our new controller [17] are confirmed by grinding experiments, especially on the view point that the force control space and the position control space are divided into orthogonal spaces being complement each other, that is, force space is defined by range space of $\boldsymbol{A}$ and the other is null space of $\boldsymbol{A}$, $(\boldsymbol{I} - \boldsymbol{A}^{+}\boldsymbol{A})$.

2. ANALYSIS OF GRINDING TASK

There are four kinds of grinding processes in common use, called respectively vertical surface grinding, horizontal surface grinding, internal grinding and cylindrical

PR0001/11/0000-0671 ¥400

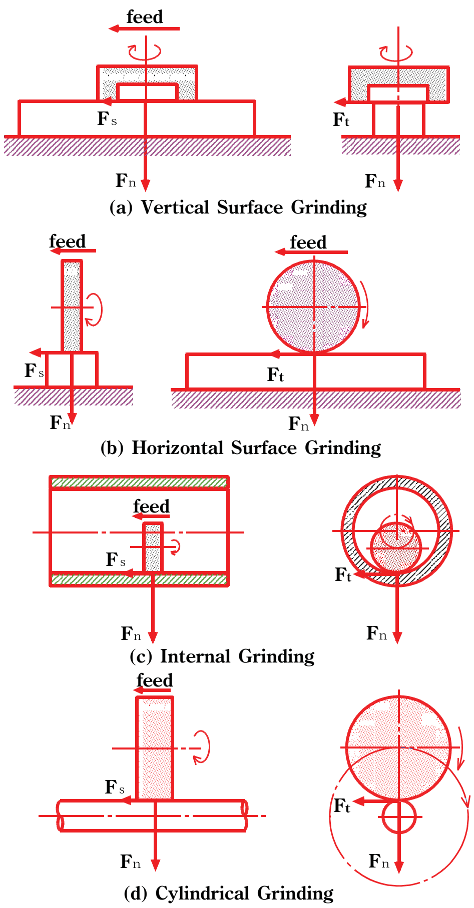

Fig. 1 Types of Grinding Tasks

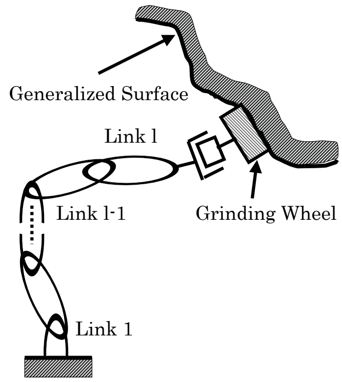

Fig. 2 A Grinding Robot

grinding. A grinding machine usually can only perform one or two of these processes because of kinematic limitation. However, all of the four kinds of tasks can be finished by a single robot manipulator for its dexterity in movement. To do so, the grinding wheel has to contact with the workpiece. A set of contacting surfaces, especially the surfaces being machined, will form constraints to the motions of the grinding wheel. As for vertical surface grinding operation shown in Fig. 1 (a) , the grinding wheel in contact with a surface of the work-piece is not free to move through that surface, which forms a position constraint. And also, the wheel cannot freely apply arbitrary force tangent to the surface in case of no disturbing force like friction existing, which forms a set of force constraint. Situations of constraints for other kinds of grinding tasks are shown in Fig. 1 (b) , (c) and (d).

Generally speaking, the grinding power is related to the metal removal rate(weight of metal being removed within unit time), which is determined by the depth of cut, the width of cut, the linear velocity of the grinding wheel, the feed rate and so on. There are many empirical formulae available for the determination of grinding power, and the desired force trajectory can then be planned according to the power. The normal grinding force F_n is exerted in the perpendicular direction of the surface. It is a significant factor that affects ground accuracy and surface roughness of workpiece. The value of it is also related to the grinding power or directly to the tangential grinding force as

$$F_t = \mathrm{K_t} F_n, \tag{2}$$

where, $\mathrm{K_t}$ is an empirical coefficient, F_t is the tangential grinding force. The axial grinding force F_n is proportional with the feed rate, and is much smaller than the former force.

Eq. (2) is based on the situation that position of the grinding cutter is controlled like currently used machining center. But when a robot is used for the grinding task, the exerting force to the object and the position of the grinding cutter should be controlled simultaneously. The F_n is generally determined by the constrained situation, and it is not suitable to apply Eq. (2) to grinding motion by the robots.

For grinding task, the normal force and tangential velocity are the most important two factors. To improve grinding quality, it is usually desired that the normal force is constant while the velocity is also constant in the middle term of a grinding stroke.

3. MODELLING

3.1 Constrained Dynamic Systems

Hemami and Wyman have addressed the issue of control of a moving robot according to constraint condition and examined the problem of the control of the biped locomotion constrained in the frontal plane. Their purpose was to control the position coordinates of the biped locomotion rather than generalized forces of constrained dynamic equation involved the item of generalized forces of constraints. And the constrained force is used as a determining condition to change the dynamic model from constrained motion to free motion of the legs. In this paper, the grinding manipulator which is shown in Fig. 2 whose end-point is in contact with the constrained surface, is modeled according Eq. (7) with Lagrangian equations of motion in term of the constraint forces, referring to what

Hemami and Arimoto have done:

$$\begin{aligned}
&\frac{d}{dt}(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}) - (\frac{\partial L}{\partial \boldsymbol{q}}) \\
= \quad & \boldsymbol{\tau} + (\frac{\partial C}{\partial \boldsymbol{q}}) / \parallel \frac{\partial C}{\partial \boldsymbol{q}} \parallel F_n - (\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}}) \dot{\boldsymbol{r}} / \parallel \boldsymbol{r} \parallel F_t \\
= \quad & \boldsymbol{\tau} + (\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}})((\frac{\partial C}{\partial \boldsymbol{r}}) / \parallel \frac{\partial C}{\partial \boldsymbol{q}} \parallel F_n - \frac{\dot{\boldsymbol{r}}}{\parallel \boldsymbol{r} \parallel} F_t) \quad (3)
\end{aligned}$$

$\boldsymbol{r}$ is the l position vector of the hand and can be expressed as a kinematic equation ,

$$\boldsymbol{r} = \boldsymbol{r}(\boldsymbol{q}). \quad (4)$$

L is the Lagrangian function, $\boldsymbol{q}$ is $l(\geq 2)$ generalized coordinates, $\boldsymbol{\tau}$ is l generalized torque inputs. The discussing robot system does not have kinematic redundancy. C is a scalar function of the constraint, and is expressed as an equation of constraints

$$C(\boldsymbol{r}(\boldsymbol{q})) = 0, \quad (5)$$

F_n is the constrained force associated with C and F_t is the tangential disturbance force.

It is easy to see that here F_n, F_t express the value of the pressure and the friction respectively, while $(\frac{\partial C}{\partial \boldsymbol{r}}) / \parallel \frac{\partial C}{\partial \boldsymbol{q}} \parallel$ and $\frac{\dot{\boldsymbol{r}}}{\parallel \boldsymbol{r} \parallel}$ express the direction of the pressure and the friction, because the friction is always vertical to the pressure, we can get

$$((\frac{\partial C}{\partial \boldsymbol{r}}) / \parallel \frac{\partial C}{\partial \boldsymbol{q}} \parallel)^T \cdot \frac{\dot{\boldsymbol{r}}}{\parallel \boldsymbol{r} \parallel} = 0 \quad (6)$$

we define $\boldsymbol{J_c}$ and $\boldsymbol{J_r}$ as

$$\boldsymbol{J}_c = \frac{\partial C}{\partial \boldsymbol{q}} / \parallel \frac{\partial C}{\partial \boldsymbol{r}} \parallel = \frac{\partial C}{\partial \boldsymbol{r}} \tilde{\boldsymbol{J}}_r / \parallel \frac{\partial C}{\partial \boldsymbol{r}} \parallel,$$

$$\tilde{\boldsymbol{J}}_r = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}}, \boldsymbol{J}_r^T = \tilde{\boldsymbol{J}}_r^T \dot{\boldsymbol{r}} / \parallel \dot{\boldsymbol{r}} \parallel,$$

(3) can be rewritten into

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}) - (\frac{\partial L}{\partial \boldsymbol{q}}) = \boldsymbol{\tau} + \boldsymbol{J}_c^T(\boldsymbol{q}) F_n - \boldsymbol{J}_r^T(\boldsymbol{q}) F_t \quad (7)$$

Eq. (7) can be derived to be

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{H}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G} = \boldsymbol{\tau} + \boldsymbol{J}_c^T(\boldsymbol{q}) F_n - \boldsymbol{J}_r^T(\boldsymbol{q}) F_t, \quad (8)$$

where $\boldsymbol{M}$ is an $l \times l$ matrix, $\boldsymbol{H}$ and $\boldsymbol{G}$ are l vectors. Because

$$\begin{aligned}
(\frac{\partial C}{\partial \boldsymbol{q}^T}) \ddot{\boldsymbol{q}} \quad &= \quad -[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C}{\partial \boldsymbol{q}^T}) \dot{\boldsymbol{q}}] \dot{\boldsymbol{q}} \\
&= \quad -\dot{\boldsymbol{q}}^T [\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C}{\partial \boldsymbol{q}^T})] \dot{\boldsymbol{q}} \quad (9)
\end{aligned}$$

And equation (8) can be rewritten as follows:

$$\begin{bmatrix} \boldsymbol{M} & -\boldsymbol{J}_c^T \\ \frac{\partial C}{\partial \boldsymbol{q}^T} & 0 \end{bmatrix} \cdot \begin{bmatrix} \ddot{\boldsymbol{q}} \\ F_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau} - \boldsymbol{H} - \boldsymbol{G} - \boldsymbol{J}_t^T F_t \\ -\dot{\boldsymbol{q}}^T [\frac{\partial C}{\partial \boldsymbol{q}}(\frac{\partial C}{\partial \boldsymbol{q}^T})] \dot{\boldsymbol{q}} \end{bmatrix} \quad (10)$$

here we define

$$\tilde{\boldsymbol{M}} = \begin{bmatrix} \boldsymbol{M} & -\boldsymbol{J}_c^T \\ \frac{\partial C}{\partial \boldsymbol{q}^T} & 0 \end{bmatrix} \quad (11)$$

in equation (10)if and only if $\tilde{\boldsymbol{M}}$ is full rank matrix we can control the manipulator to the desired pose. Here we express $\tilde{\boldsymbol{M}}$ in a simple form:

$$\tilde{\boldsymbol{M}} = \begin{bmatrix} \boldsymbol{M} & -l\boldsymbol{A}^T \\ \boldsymbol{A} & 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{M} & -\boldsymbol{J}_c^T \\ \frac{\partial C}{\partial \boldsymbol{q}^T} & 0 \end{bmatrix} \quad (12)$$

here $l = 1 / \parallel \frac{\partial C}{\partial \boldsymbol{q}} \parallel > 0$, because $\boldsymbol{M}$ is positive definite, there exist a orthogonal matrix $\boldsymbol{V}$ which can makes

$$\boldsymbol{V}\boldsymbol{M}\boldsymbol{V}^T = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{bmatrix} (\lambda_1 \dots \lambda_n > 0) \quad (13)$$

so

$$\begin{aligned}
det(\tilde{\boldsymbol{M}}) \quad &= \quad det(\boldsymbol{M}^{-1}) det(\boldsymbol{A}\boldsymbol{M}l\boldsymbol{A}^T) \\
&= \quad l \cdot det(\boldsymbol{M}^{-1}) det(\boldsymbol{A}\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{M}\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{A}^T) \quad (14)
\end{aligned}$$

define

$$\boldsymbol{A}\boldsymbol{V}^T = [a_1, \dots, a_n] \quad (15)$$

so

$$det(\tilde{\boldsymbol{M}}) = l \cdot det(\boldsymbol{M}) \sum_{i=1}^{n} (\lambda_i a_i^2) > 0 \quad (16)$$

which means that $\tilde{\boldsymbol{M}}$ is reversible.

The state variable $\boldsymbol{x}$ is constructed by adjoining $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$: $\boldsymbol{x} = (\boldsymbol{x}_1^T, \boldsymbol{x}_2^T)^T$=$(\boldsymbol{q}^T, \dot{\boldsymbol{q}}^T)^T$. The state-space equation of the system are

$$\begin{aligned}
\dot{\boldsymbol{x}}_1 \quad &= \quad \boldsymbol{x}_2, \\
\dot{\boldsymbol{x}}_2 \quad &= \quad -\boldsymbol{M}^{-1}(\boldsymbol{H}(\boldsymbol{x}_1, \boldsymbol{x}_2) + \boldsymbol{G}(\boldsymbol{x}_1)) \\
& \qquad + \boldsymbol{M}^{-1}(\boldsymbol{\tau} + \boldsymbol{J}_c^T(\boldsymbol{x}_1) F_n - \boldsymbol{J}_r^T \boldsymbol{x}_1) F_t) \quad (17)
\end{aligned}$$

or in the compact form

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{\tau}, F_n, F_t), \quad (18)$$

Using the inverted form of combination from Eq. (5) and Eq.(18) (this part had been detailedly introduced in [19] by us), F_n can be expressed as

$$F_n = F_n(\boldsymbol{x}, \boldsymbol{\tau}, F_t), \quad (19)$$

or in a more detailed form

$$\begin{aligned}
F_n = & [(\frac{\partial C}{\partial \boldsymbol{q}}) \boldsymbol{M}^{-1} (\frac{\partial C}{\partial \boldsymbol{q}})^T]^{-1} \parallel \frac{\partial C}{\partial \boldsymbol{r}} \parallel \\
& \{-[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C}{\partial \boldsymbol{q}}) \dot{\boldsymbol{q}}] \dot{\boldsymbol{q}} + (\frac{\partial C}{\partial \boldsymbol{q}}) \boldsymbol{M}^{-1} (\boldsymbol{H}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) + \boldsymbol{J_r}^T F_t)\} \\
& -[(\frac{\partial C}{\partial \boldsymbol{q}}) \boldsymbol{M}^{-1} (\frac{\partial C}{\partial \boldsymbol{q}})^T]^{-1} \parallel \frac{\partial C}{\partial \boldsymbol{r}} \parallel \{(\frac{\partial C}{\partial \boldsymbol{q}}) \boldsymbol{M}^{-1}\} \boldsymbol{\tau} \\
& \triangleq a(\boldsymbol{x}_1, \boldsymbol{x}_2) + \boldsymbol{A}(\boldsymbol{x}_1) \boldsymbol{J}_r^T F_t - \boldsymbol{A}(\boldsymbol{x}_1) \boldsymbol{\tau}, \quad (20)
\end{aligned}$$

where, $a(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is a scalar representing the first term in the expression of F_n, and $\boldsymbol{A}(\boldsymbol{x}_1)$ is an l vector to represent the coefficient vector of $\boldsymbol{\tau}$ in the same expression. Eq. (18) and Eq. (19) compose a constrained system that can be controlled, if $F_n = 0$, describing the unconstrained motion of the system.

Substituting Eq. (20) into Eq. (17), the state equation of the system including the constrained force (as $F_n > 0$) can be rewritten as

$$\begin{aligned}
\dot{\boldsymbol{x}}_1 &= \boldsymbol{x}_2, \\
\dot{\boldsymbol{x}}_2 &= -\boldsymbol{M}^{-1}[\boldsymbol{H}(\boldsymbol{x}_1, \boldsymbol{x}_2) + \boldsymbol{G}(\boldsymbol{x}_1) - \boldsymbol{J}_c^T(\boldsymbol{x}_1)a(\boldsymbol{x}_1, \boldsymbol{x}_2)] \\
&\quad +\boldsymbol{M}^{-1}[(\boldsymbol{I} - \boldsymbol{J}_c^T\boldsymbol{A})\boldsymbol{\tau} + (\boldsymbol{J}_c^T\boldsymbol{A} - \boldsymbol{I})\boldsymbol{J}_r^T F_t], \quad (21)
\end{aligned}$$

Solutions of these dynamic equation always satisfy the constrained condition (5).

4. FORCE AND POSITION CONTROLLER WITH LYAPUNOV STABILITY

4.1 Preparation for the proof

From Lagrange equation we can get the dynamics equation of the manipulator

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{q}}(\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau} \quad (22)$$

here $\boldsymbol{M}(\boldsymbol{q})$ is the inertial moment matrix, $\boldsymbol{q}$ express the angle of the joints, $\boldsymbol{g}(\boldsymbol{q})$ is the gravity term. The energy of a manipulator system is:

$$\begin{aligned}
E &= K + U \quad (23)\\
&= \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + U(q) \quad (24)
\end{aligned}$$

here, E is the whole energy of the system, K is kinetic energy and U is the gravity potential energy, so

$$\begin{aligned}
\dot{E} &= \frac{1}{2}\ddot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{\partial U}{\partial \boldsymbol{q}^T}\dot{\boldsymbol{q}} \\
&= \dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\frac{\partial U}{\partial \boldsymbol{q}^T} \quad (25)\\
&= \dot{\boldsymbol{q}}^T\big(\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{\partial U}{\partial \boldsymbol{q}^T}\big) \quad (26)
\end{aligned}$$

from (22) and (26) we can get

$$\begin{aligned}
&\dot{\boldsymbol{q}}^T\big(\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q})\big) \quad (27)\\
&= -\dot{\boldsymbol{q}}^T\Big(\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{q}^T}\big(\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\big)\Big) \quad (28)\\
&= -\frac{1}{2}\dot{\boldsymbol{q}}^T\Big(\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} - \frac{\partial}{\partial \boldsymbol{q}^T}\big(\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\big)\Big)\dot{\boldsymbol{q}} \quad (29)
\end{aligned}$$

set

$$\boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\Big(\dot{\boldsymbol{M}}(\boldsymbol{q}) - \frac{\partial}{\partial \boldsymbol{q}^T}\big(\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}\big)\Big) \quad (30)$$

so (29) can be simplified to

$$\dot{\boldsymbol{q}}^T\big(\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q})\big) = -\dot{\boldsymbol{q}}^T\boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \quad (31)$$

It is easy to see that $\boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is a skew-symmetrical matrix, and the manipulator dynamics equation (22) can be written as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau}. \quad (32)$$

4.2 Force and position controller with Lyapunov Stability

We make the input torque as:

$$\begin{aligned}
\boldsymbol{\tau} = -\boldsymbol{A}^{+}(x_1)\{F_{nd} - \boldsymbol{a}(x_1, x_2) - \boldsymbol{A}(x_1)\boldsymbol{J}_r^T F_t\} \\
-(I - \boldsymbol{A}^{+}(x_1)\boldsymbol{A}(x_1))\boldsymbol{k} \quad (33)
\end{aligned}$$

here we define $\boldsymbol{N} = \boldsymbol{I} - \boldsymbol{A}^{+}\boldsymbol{A}$, and where $\boldsymbol{I}$ is a $l \times l$ identity matrix, F_{nd} is the desired constrained forces, $\boldsymbol{A}(x_1)$ is defined in Eq. (20) and $\boldsymbol{A}^{+}(x_1)$ is the pseudoinverse matrix of it, $a(x_1, x_2)$ is also defined in Eq. (20) and $\boldsymbol{k}$ is an arbitrary vector which is defined as

$$\boldsymbol{k} = \boldsymbol{J}_c^T F_n - \boldsymbol{J}_r^T F_t - \boldsymbol{c}[k_p(r_x - r_{xd}) + k_d\dot{r}_x] \quad (34)$$

because $\boldsymbol{V}^T\boldsymbol{N} \neq \boldsymbol{0}$, there always exist a vector $\boldsymbol{c}$ make $\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{c} \neq 0$, here $\boldsymbol{c}$ is a arbitrary vector which satisfies $\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{c} \neq 0$. The position of the end-effector can be expressed as $\boldsymbol{r} = [r_x, r_y, r_z]$ so,

$$\dot{\boldsymbol{q}} = \boldsymbol{V}\dot{r}_x \quad (35)$$

here, $\boldsymbol{V}$ is a 3×1 vector, differentiate both side of (35) by time

$$\ddot{\boldsymbol{q}} = \dot{\boldsymbol{V}}\dot{r}_x + \boldsymbol{V}\ddot{r}_x \quad (36)$$

here we do not consider the gravity and friction so the dynamics equation of the system can be simplified to:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{H}\dot{\boldsymbol{q}} = \boldsymbol{\tau} + \boldsymbol{J}_c^T F_n - \boldsymbol{J}_r^T(\boldsymbol{q})F_t \quad (37)$$

take (35) and (36) into (37) we can get

$$\begin{aligned}
&\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{V}}\dot{r}_x + \boldsymbol{M}(\boldsymbol{q})\boldsymbol{V}\ddot{r}_x + \boldsymbol{H}\boldsymbol{V}\dot{r}_x \\
&= \boldsymbol{\tau} + \boldsymbol{J}_c^T F_n - \boldsymbol{J}_r^T(\boldsymbol{q})F_t \quad (38)
\end{aligned}$$

premultipuly $\boldsymbol{V}^T\boldsymbol{N}$ on both side we can get

$$\begin{aligned}
&\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{V}}\ddot{r}_x + \boldsymbol{V}^T\boldsymbol{N}(\boldsymbol{M}(\boldsymbol{q}))\boldsymbol{V}\dot{r}_x \\
&+\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{H}\boldsymbol{V}\dot{r}_x \\
&= \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{\tau} + \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{J}_c^T F_n - \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{J}_r^T(\boldsymbol{q})F_t \quad (39)
\end{aligned}$$

Take (33) into (39) we can get

$$\begin{aligned}
&\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{V}}\ddot{\boldsymbol{r}}_x + \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{V}\dot{r}_x + \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{H}\boldsymbol{V}\dot{r}_x \\
&= \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{k} + \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{J}_c^T F_n - \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{J}_r^T(\boldsymbol{q})F_t \quad (40)\\
&= \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{c}(-k_d\dot{r}_x - k_p(r_x - r_{xd})) \quad (41)\\
&= -k_d\dot{r}_x - k_p(r_x - r_{xd}) \quad (42)
\end{aligned}$$

$\boldsymbol{H}$ can be divided into $\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S}$ here $\boldsymbol{S}$ is an antisymmetric matrix, so (40) can be written as:

$$\begin{aligned}
&\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{V}}\ddot{r}_x + \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{V}}\dot{r}_x \\
&+\boldsymbol{V}^T\boldsymbol{N}(\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S})\boldsymbol{V}\dot{r}_x + k_d\dot{r}_x + k_p(r_x - r_{xd}) \\
&= 0 \quad (43)
\end{aligned}$$

multiple $\dot{r}_x$ on the both side of (43) we can get:

$$\begin{aligned}
&\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{V}\dot{r}_x\ddot{r}_x + \boldsymbol{V}^T\boldsymbol{N}(\boldsymbol{M}(\boldsymbol{q}))\dot{\boldsymbol{V}}\dot{r}_x^2 \\
&+\boldsymbol{V}^T\boldsymbol{N}(\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{S})\boldsymbol{V}\dot{r}_x^2 + k_d\dot{r}_x^2 + k_p(r_x - r_{xd})\dot{r}_x \\
&= 0 \quad (44)
\end{aligned}$$

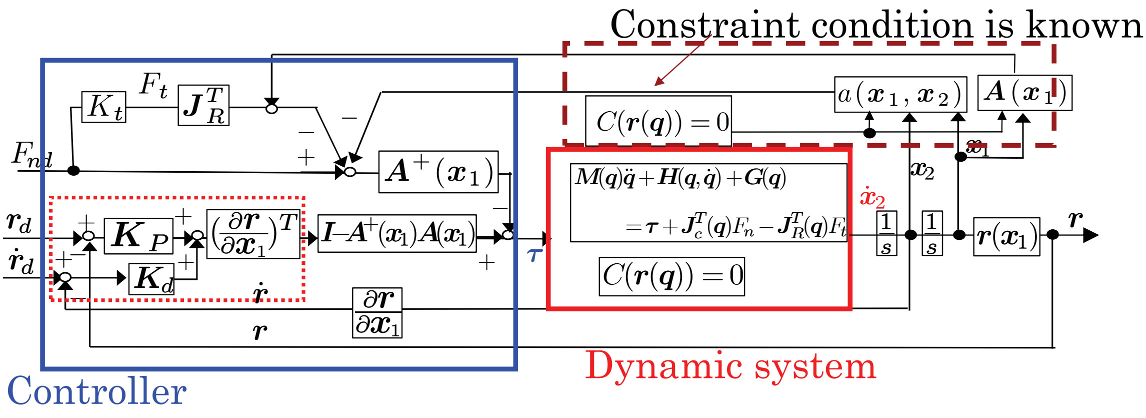

Fig. 3 Shape-grinding position / force control system

set Lyapunov argument as:

$$\begin{aligned} V &= \frac{1}{2}\boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{V}\dot{r}_x^2 + \int_0^t \frac{1}{2}\boldsymbol{V}^T\boldsymbol{A}^+\boldsymbol{A}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{V}\dot{r}_x^2 dt \\ &\quad + \frac{1}{2}k_p(r_x - r_{xd})^2 \geq 0 \end{aligned} \tag{45}$$

so

$$\begin{aligned} \dot{V} &= \boldsymbol{V}^T\boldsymbol{N}\boldsymbol{M}(\boldsymbol{q})\boldsymbol{V}\dot{r}_x\ddot{r}_x + \boldsymbol{V}^T\boldsymbol{N}(\boldsymbol{M}(\boldsymbol{q}))\dot{\boldsymbol{V}}\dot{r}_x^2 \\ &\quad + \boldsymbol{V}^T\boldsymbol{N}\frac{1}{2}\dot{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{V}\dot{r}_x^2 + k_p(r_x - r_{xd})\dot{r}_x \end{aligned} \tag{46}$$

from (44), (46) can be transformed to

$$\begin{aligned} \dot{V} &= -\boldsymbol{V}^T\boldsymbol{S}\boldsymbol{V}\dot{r}_x^2 - k_d\dot{r}_x^2 \\ &= -k_d\dot{r}_x^2 \leq 0 \end{aligned} \tag{47}$$

because $\boldsymbol{S}$ is a skew symmetrical matrix. From (46) we can see only when $\dot{r}_x = 0$, $\dot{V} = 0$, in this case we assume that $r_x \neq r_{xd}$ so from (33) we can know that $\boldsymbol{\tau} \neq \boldsymbol{0}$ this conflict to $\dot{r}_x = 0$ so we can say that if and only if $r_x = r_{xd}$, $\dot{r}_x = 0$ and $\dot{V} = 0$, and then get the conclusion:

$$\lim_{t\to\infty} r_x = r_{xd} \tag{48}$$

because the constraint system satisfies that $C(\boldsymbol{r}(\boldsymbol{q})) = 0$,

$$\lim_{t\to\infty} \boldsymbol{r} = \lim_{t\to\infty} \boldsymbol{r}_d. \tag{49}$$

Because (20) is a function does not affected by time, when we substitute (34) into (20), we can get

$$F_n = F_{nd} \tag{50}$$

here (50) does not conclude the variable of time which means that, the output force always equals the desired one.

5. CONCLUSIONS

In this paper we designed a constraint-combined force/position controller for the continuous shape-grinding system, and prove the convergence of the controller in a new way by Lyapunov method, the output force of the system can always equel to the desired once. At last we did some simulations to verify the convergence of the controller.

REFERENCES

[1] M.H.Raibert and J.J.Craig: Hybrid Position/Force Control of Manipulators, Trans. of the ASME, J. of Dynamic Systems, Measurement and Control, Vol.102, pp.126-133, June, 1981.

[2] S. Arimoto, Mechanics and Control of Robot(in Japanese), Asakura Publishing Co., Ltd., Tokyo, Japan, 1990.

[3] T. Yoshikawa: Dynamic Hybrid Position/Force control of Robot Manipulators — Description of Hand Constraints and Calculation of Joint Driving Force, IEEE J. on Robotics and Automation, Vol.RA-3, No.5, pp.386-392, 1987.

[4] L. Whitcomb, S. Arimoto, T. Naniwa and F. Osaki, Experiments in Adaptive Model-Based Force Control, IEEE Control Systems Society, Vol.16, No.1, pp.49-57, 1996.

[5] S. Arimoto: Joint-Space Orthogonalization and Passivity for Physical Interpretations of Dextrous Robot Motions under Geometric Constrains, Int. J. of Robust and Nonlinear Control, Vol.5, pp.269-284, 1995.

[6] D.Wang and N.H. McClamroch: Position and Force Control for Constrained Manipulator Motion: Lyapunov's Direct Approach, IEEE Trans. Robotics and Automation, Vol.9, pp.308-313, 1993.

[7] Z. X. Peng and N. Adachi: Position and Force Control of Manipulators without Using Force Sensors(in Japanese), Trans. of JSME(C), Vol.57, No.537, pp.1625-1630, 1991.

[8] Jaydeep. Roy and Louis L. Whitcomb: Adaptive Force Control of Position/Velocity Controlled Robots: Theory and Experiment, IEEE Transactions on Robotics and Automation, Vol.18, No.2, April, 2002.

[9] Suguru Arimoto, Control Theory of Non-Linear Mechanical Systems, Oxford University Press, 1996.

[10] B. Siciliano, L. Villani, Robot Force Control, Kluwer Academic Publishers, U.K., 1999.

[11] C. Natale, Interaction Control of Robot Manipulators, Springer Tracts in Advanced Robotics, Germany, 2003.

[12] N. Hogan: Impedance control: An approach to manipulation: Part 1-Theory, J.Dynamic Syst., Measurement Contr., Vol.107, pp.1-7, Mar, 1985.
[13] B. Siciliano and L. Villani: A passivity-based approach to force regulation and motion control of robot manipulators, Automatica, Vol.32, No.3,pp.443-447, Mar, 1996.
[14] L. Villani, C.C. de Wit and B. Brogliato: An exponentially stable adaptive control for force and position tracking of robot manipulators, IEEE Trans. Automat. Contr., Vol44, pp.778-802, Apr, 1999.
[15] J. De Schutter and H.Van Brussel: Compliant robot motion 2. A control approach based on external control loops, Int. J. Robot. Res., Vol.7, No.4, pp.18-33, Aug, 1988.
[16] Hooshang Hemami and Bostwick F. Wyman: Modeling and Control of Constrained Dynamic Systems with Application to Biped Locomotion in the Frontal Plane, IEEE Trans. on Automatic Control, Vol.AC-24, No.4, pp.526-535, 1979.
[17] Takeshi Ikeda, Mamoru Minami, Position/Force Control of a Manipulator by Using an Algebraic Relation and Evaluations by Experiments, The 2003 IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics (AIM2003), Proceeding CD-ROM paper No.159, 503-508, 2003
[18] Takeshi Ikeda and Mamoru Minami: Research of Grinding Robot without Force Sensor by Using Algebraic Equation(in Japanese), Transactions of the Japan Society of Mechanical Engineers(C), Vol.71, No.702, pp.270-277, 2005.
[19] Mamoru Minami, Weiwei Xu, Shape-grinding by Direct Position / Force Control with On-line Constraint Estimation, 2008 IEEE/RSJ International Conference on Intelligent Robots and Systems, Acropolis Convention Center, Nice, France, Sept, 22-26, 2008. pp. 943-948