SICE Annual Conference 2011
September 13-18, 2011, Waseda University, Tokyo, Japan

Application of Strongly Stable Generalized Predictive Control to Temperature Control of an Aluminum Plate

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Abstract: This paper explores an application of strongly stable generalized predictive control (GPC) to temperature control of an aluminum plate. The proposed method gives the same output response as the closed-loop response of GPC designed in advance, even if the feedback loop is cut. The effectiveness of the proposed method is verified by applying to temperature control of an aluminum plate.

Keywords: Strongly stable, Generalized predictive control, Temperature control

1. INTRODUCTION

This paper proposes a design method of strongly stable generalized predictive control (GPC) focused on closed-loop characteristics. GPC technique is first proposed by Clarke and others in 1987 [1]. The control method has features that the objective function includes prediction and control horizons, and control signals are computed by receding horizons. With these features, the control strategy has been accepted by many of practical engineers and applied widely in industry [2].

In applying to industry, safety is most important. For safety, it is desirable to design a controller itself to be stable in addition to the closed-loop stability, that is, a strongly stable controller is needed. The strongly stable controller keeps the boundedness of control input even when the feedback loop is cut by an accident. So far many papers concerning on the stability of GPC have been published, but most of the papers are devoted to obtain design methods ensuring the stability of closed-loop system and several design schemes have been established [3][4], although less attention is paid to the design of strongly stable GPC.

Hence to obtain a strongly stable GPC, we can use GPC with the closed-loop stability designed by an existing method. And in this paper we will propose a method to design a strongly stable GPC by coprime factorization [5]. GPC can be extended by coprime factorization, and the extended controller can be designed to be stable by selecting newly introduced parameter [6][7]. That is, strongly stable system, which means both the closed-loop system and its controller are stable, can be obtained. Although the authors have considered the design method of strongly stable system using coprime factorization, the steady state of output has not been considered when feedback loop was cut. In the case that the controlled plant is stable, the steady state of output of strongly stable system is stable even if feedback loop is cut. But for safety, it is desirable that the steady state of output becomes as close to the steady state of closed-loop output as possible even if feedback loop was cut. Therefore this paper explores a design method of strongly stable GPC focused on closed-loop characteristics by algebraic calculation of newly introduced parameter in the extended GPC. The proposed method has the feature that the steady state of output becomes the same as the steady state of closed-loop output even if feedback loop was cut.

2. PROBLEM STATEMENT

Consider a single-input single-output system:

$$A[z^{-1}]y(t) = z^{-k_m}B[z^{-1}]u(t), \tag{1}$$

where $y(t)$ and $u(t)$ denote output and input, k_m is time delay. $A[z^{-1}]$ and $B[z^{-1}]$ are n-order and m-order polynomials respectively. Firstly the prediction of $y(t)$ is considered for the deviation system of the plant Eq. (1). The steady state values y_∞ of $y(t)$ and u_∞ of $u(t)$ are derived as follows.

$$A[z^{-1}]y_\infty = z^{-k_m}B[z^{-1}]u_\infty \tag{2}$$

From this equation, the deviation system of the plant Eq.(1) is obtained.

$$A[z^{-1}]\tilde{y}(t) = z^{-k_m}B[z^{-1}]\tilde{u}(t) \tag{3}$$

Where the deviations $\tilde{y}(t)$ and $\tilde{u}(t)$ are defined as $\tilde{y}(t) = y(t) - y_\infty$ and $\tilde{u}(t) = u(t) - u_\infty$. Assuming that y_∞ is equal to be the reference signal w, the prediction for Eq. (3) can be derived by the following Diophantine equation. To separate the future values and past values of $u(t)$ in Eq. (3), $E_j[z^{-1}]B[z^{-1}]$ is separated as

$$1 = A[z^{-1}]E_j[z^{-1}] + z^{-j}F_j[z^{-1}],$$
$$E_j[z^{-1}]B[z^{-1}] = R_j[z^{-1}] + z^{-j}S_j[z^{-1}],$$

where $E_j[z^{-1}], F_j[z^{-1}]$ are polynomials with degree $j-1$ and $n-1$, and $E_j[z^{-1}]$ is monic. $R_j[z^{-1}]$ and $S_j[z^{-1}]$ are polynomials with degree of $j-1$ and $n_3 = \max\{m, l\} - 1$.

$$\begin{aligned} E_j[z^{-1}] &= 1 + e_1 z^{-1} + ... + e_{j-1}z^{-(j-1)}, \\ F_j[z^{-1}] &= f_0^j + f_1^j z^{-1} + ... + f_{n-1}^j z^{-(n-1)}, \\ R_j[z^{-1}] &= r_0 + r_1 z^{-1} + ... + r_{j-1}z^{-(j-1)}, \\ S_j[z^{-1}] &= s_0 + s_1^j z^{-1} + ... + s_{m_3}^j z^{-n_3}. \end{aligned}$$

In order to find j-ahead prediction $\hat{\tilde{y}}(t+j|t)$, j-ahead output $\tilde{y}(t+j)$ is derived as,

$$\tilde{y}(t+j) = R_j[z^{-1}]\tilde{u}(t+j-k_m) + h_j(t),$$

where

$$h_j(t) = F_j[z^{-1}]\tilde{y}(t) + S_j[z^{-1}]\tilde{u}(t-k_m).$$

Because it is assumed that there exists no perturbation, j-ahead prediction is given as $\hat{\tilde{y}}(t+j|t) = \tilde{y}(t+j)$. The performance index for the deviation system Eq. (3) is considered.

$$J = \sum_{j=N_1}^{N_2}\{\tilde{y}(t+j)\}^2 + \lambda\sum_{j=1}^{Nu}\{\tilde{u}(t+j-1)\}^2 \quad (4)$$

N_1 and N_2 are prediction horizon, N_u is control horizon and λ_j $(j = 1, \cdots, N_2)$ is weighting factor on control inputs. For simplicity it is assumed that $N_1 = k_m = 1, N_u = N_2$.

Minimizing the performance index J on $\tilde{\boldsymbol{u}}$, the control law is derived. Here we define $\hat{\tilde{\boldsymbol{y}}}$, $\tilde{\boldsymbol{u}}$, $\boldsymbol{h}$, and matrix $\boldsymbol{R}$.

$$\hat{\tilde{\boldsymbol{y}}} = [\tilde{y}(t+1|t), \tilde{y}(t+2|t), \cdots, \tilde{y}(t+N_2|t)]^T \quad (5)$$

$$\tilde{\boldsymbol{u}} = [\tilde{u}(t), \tilde{u}(t+1), \cdots, \tilde{u}(t+N_2-1)]^T \quad (6)$$

$$\boldsymbol{h} = [h_1(t), h_2(t), \cdots, h_{N_2}(t)]^T \quad (7)$$

$$\boldsymbol{R} = \begin{bmatrix} r_0 & 0 & \cdots & & 0 \\ r_1 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ r_{N_2-1} & \cdots & \cdots & r_1 & r_0 \end{bmatrix}$$

Then the prediction $\hat{\tilde{\boldsymbol{y}}}$ and the objective function J of Eq. (4) are given as vector form:

$$\hat{\tilde{\boldsymbol{y}}} = \boldsymbol{R}\tilde{\boldsymbol{u}} + \boldsymbol{h} \quad (8)$$

$$\begin{aligned} J &= \hat{\tilde{\boldsymbol{y}}}^T\hat{\tilde{\boldsymbol{y}}} + \lambda\tilde{\boldsymbol{u}}^T\tilde{\boldsymbol{u}} \\ &= (\boldsymbol{R}\tilde{\boldsymbol{u}} + \boldsymbol{h})^T(\boldsymbol{R}\tilde{\boldsymbol{u}} + \boldsymbol{h}) + \lambda\tilde{\boldsymbol{u}}^T\tilde{\boldsymbol{u}} \end{aligned} \quad (9)$$

Solving the linear equation $\partial J/\partial\hat{\boldsymbol{u}}= 0$ of $\tilde{\boldsymbol{u}}$ and extracting the first element of $\tilde{\boldsymbol{u}}$, the control input $u(t)$ to minimize J is obtained as

$$\begin{aligned} u(t) &= \frac{F_p[z^{-1}] + (1+z^{-1}S_p[z^{-1}])K}{1+z^{-1}S_p[z^{-1}]}w \\ &\quad -\frac{F_p[z^{-1}]}{1+z^{-1}S_p[z^{-1}]}y(t), \end{aligned} \quad (10)$$

where

$$[p_1, \cdots, p_{N_2}] = -[1, 0, \cdots, 0](\boldsymbol{R}^T\boldsymbol{R} + \lambda\boldsymbol{I})^{-1}\boldsymbol{R}^T,$$

$$F_p[z^{-1}] = \sum_{j=1}^{N_2} p_jF_j[z^{-1}], \quad K = \frac{A[1]}{B[1]},$$

$$S_p[z^{-1}] = \sum_{j=1}^{N_2} p_jS_j[z^{-1}].$$

Then the closed-loop system is given by

$$y(t) = \frac{z^{-1}B[z^{-1}]\{F_p[z^{-1}] + (1+z^{-1}S_p[z^{-1}])K\}}{T[z^{-1}]}w, \quad (11)$$

where $T[z^{-1}] = A[z^{-1}] + z^{-1}D_p[z^{-1}]$ and $D_P[z^{-1}] = A[z^{-1}]S_p[z^{-1}] + B[z^{-1}]F_p[z^{-1}]$.

3. PROPOSED METHOD

For coprime factorization approach, the family of stable rational functions is considered.

$$RH_\infty = \{G(z^{-1}) = \frac{G_n[z^{-1}]}{G_d[z^{-1}]}\},$$

where $G_d[z^{-1}]$ is stable polynomial. Transfer function of the plant is given in the form of a ratio of rational functions in RH_∞,

$$G(z^{-1}) = N(z^{-1})/D(z^{-1}) \quad (12)$$

$$y(t) = G(z^{-1})u(t) = N(z^{-1})D^{-1}(z^{-1})u(t) \quad (13)$$

where $N(z^{-1})$ and $D(z^{-1})$ are rational functions in RH_∞ and coprime each other. Then all the stabilizing compensator is given in Youla-Kucera parameterization.

$$\begin{aligned} u(t) &= C_1(z^{-1})w - C_2(z^{-1})y(t) \quad (14) \\ C_1(z^{-1}) &= (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}K(z^{-1}) \\ C_2(z^{-1}) &= (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1} \\ &\quad \cdot(X(z^{-1}) + U(z^{-1})D(z^{-1})) \end{aligned}$$

where $U(z^{-1})$ and $K(z^{-1})$ are rational functions in RH_∞ and are design parameters. $X(z^{-1})$ and $Y(z^{-1})$ are also in RH_∞ and are the solutions of the following Bezout equation.

$$X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1 \quad (15)$$

If $T[z^{-1}]$ in Eq. (11) is designed to be stable, comparing Eq. (1) to Eq. (13), $N(z^{-1})$ and $D(z^{-1})$ can be chosen as follows;

$$N(z^{-1}) = \frac{z^{-1}B[z^{-1}]}{T[z^{-1}]}, \; D(z^{-1}) = \frac{A[z^{-1}]}{T[z^{-1}]}$$

$X(z^{-1})$ and $Y(z^{-1})$ can be given as

$$X(z^{-1}) = F_p[z^{-1}], \; Y(z^{-1}) = 1 + z^{-1}S_p[z^{-1}]$$

Designing $K(z^{-1}) = F_p[z^{-1}] + (1+z^{-1}S_p[z^{-1}])K$ and $U(z^{-1}) = 0$, Eq. (14) expresses the controller Eq. (10). And the closed-loop transfer function Eq. (11) is also expressed as,

$$y(t) = N(z^{-1})K(z^{-1})w \quad (16)$$

In order to extend the controller Eq. (10) by using $U(z^{-1}) \neq 0$, this paper considers the steady state of system in the case that feedback loop is cut by accident. When the feedback loop is cut, the relation of the reference signal and the output is given as follows.

$$D(z^{-1})(Y(z^{-1}) - U(z^{-1})N(z^{-1}))y(t) = N(z^{-1})K(z^{-1})w$$

To make $y(t)$ track to w in the steady state, the design parameter is newly designed as,

$$U(z^{-1}) = -D^{-1}(1)X(1) \quad (17)$$

By using the above $X(z^{-1})$, $Y(z^{-1})$, $N(z^{-1})$, $D(z^{-1})$, $K(z^{-1})$ and $U(z^{-1})$, the new controller is obtained. And the closed-loop transfer function is equal to Eq. (16). Moreover, in the case that the strongly stable system is obtained, the system output can track to the reference signal even if the feedback loop is cut.

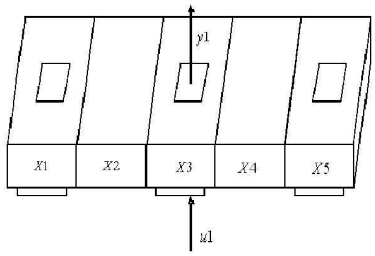

Fig. 1 The aluminum plate thermal process

4. ALUMINUM PLATE MODEL

When the aluminum plate thermal process is transformed into mathematical model, elicitation process of state equations is shown in following. State variables are set like this form

$$\begin{aligned} x_1 &= T_1 - T_6 \\ x_2 &= T_2 - T_6 \\ x_3 &= T_3 - T_6 \\ x_4 &= T_4 - T_6 \\ x_5 &= T_5 - T_6 \end{aligned}$$

T_1, T_2, T_3, T_4, T_5, and T_6 are temperature in each part of the almuminum plate and ambient temperature. Three laws were used in the development of the mathematical model.
Fourier's law of heat conduction is given as,

$$q[W/m^2] = -\lambda[W/mK](d\theta/dn)[K/m]$$

where q is heat flow ratio, λ is coefficient of thermal conductivity, $(d\theta/dn)$ is temperature cant of heat current.
Newton's law of cooling is given as,

$$q[W/m^2] = \alpha[W/m^2K](\theta_s - \theta_f)[K]$$

where α is coefficient of thermal conductivity(current:10 $\sim$ 600$[W/m^2K]$). Equation among heat capacity, objects and their specific heat is,

$$dQ[J] = c[J/kgK]m[kg]d\theta[K]$$

where c is specific heat and m is mass. From these laws, lumped parameter system is given. Each part of the system is

$$\begin{aligned} mc\frac{d(T_1 - T_6)}{dt} &= -(\alpha(T_1 - T_6)(S_1 + S_2 + 2S_3) \\ &\quad +\lambda\frac{T_1 - T_2}{d_2}S_2) \\ mc\frac{d(T_2 - T_6)}{dt} &= -(\alpha(T_2 - T_6)(2S_4 + 2S_5) \\ &\quad +\lambda\frac{T_2 - T_1}{d_3}S_2 + \lambda\frac{T_2 - T_3}{d_3}S_2) \\ mc\frac{d(T_3 - T_6)}{dt} &= u_1 - (\alpha(T_3 - T_6)(S_1 + 2S_3) \\ &\quad +\lambda\frac{T_3 - T_2}{d_2}S_2 + \lambda\frac{T_3 - T_4}{d_2}S_2) \\ mc\frac{d(T_4 - T_6)}{dt} &= -(\alpha(T_4 - T_6)(2S_4 + 2S_5) \\ &\quad +\lambda\frac{T_4 - T_3}{d_3}S_2 + \lambda\frac{T_4 - T_5}{d_3}S_2) \\ mc\frac{d(T_5 - T_6)}{dt} &= -(\alpha(T_5 - T_6)(S_1 + S_2 + 2S_3) \\ &\quad +\lambda\frac{T_5 - T_4}{d_2}S_2) \end{aligned}$$

Therefore

$$\begin{aligned} mc\frac{dx_1}{dt} &= -(\alpha(S_1 + S_2 + 2S_3) + \lambda\frac{S_2}{d_2})x_1 + (\lambda\frac{S_2}{d_2})x_2 \\ mc\frac{dx_2}{dt} &= (\lambda\frac{S_2}{d_3})x_1 - (\alpha(2S_4 + 2S_5) + \lambda\frac{S_2}{d_3})x_2 \\ &\quad +(\lambda\frac{S_2}{d_3})x_3 \\ mc\frac{dx_3}{dt} &= (\lambda\frac{S_2}{d_2})x_2 - (\alpha(S_1 + 2S_3) + \lambda\frac{S_2}{d_2})x_3 \\ &\quad +(\lambda\frac{S_2}{d_2})x_4 + u_1 \\ mc\frac{dx_4}{dt} &= (\lambda\frac{S_2}{d_3})x_3 - (\alpha(2S_4 + 2S_5) + \lambda\frac{S_2}{d_3})x_4 \\ &\quad +(\lambda\frac{S_2}{d_3})x_5 \\ mc\frac{dx_5}{dt} &= (\lambda\frac{S_2}{d_2})x_4 - (\alpha(S_1 + S_2 + 2S_3) + \lambda\frac{S_2}{d_2})x_5 \end{aligned}$$

From these equations, state equation of single input single output five degree prosess is obtained.

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) \end{aligned} \tag{18}$$

$$\boldsymbol{A} = \frac{1}{mc} \cdot \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{bmatrix}$$

where

$$\begin{aligned} a_{11} &= -(\alpha(S_1 + S_2 + 2S_3) + \lambda\frac{S_2}{d_2}) \\ a_{12} &= \lambda\frac{S_2}{d_2}, a_{21} = -\lambda\frac{S_2}{d_3} \\ a_{22} &= -(\alpha(2S_4 + 2S_5) + \lambda\frac{S_2}{d_3}) \\ a_{23} &= \lambda\frac{S_2}{d_3}, a_{32} = \lambda\frac{S_2}{d_2} \\ a_{33} &= -(\alpha(S_1 + 2S_3) + \lambda\frac{S_2}{d_2}) \\ a_{34} &= \lambda\frac{S_2}{d_2}, a_{43} = \lambda\frac{S_2}{d_3} \\ a_{44} &= -(\alpha(2S_4 + 2S_5) + \lambda\frac{S_2}{d_3}) \end{aligned}$$

$$a_{45} = \lambda\frac{S_2}{d_3}, a_{54} = \lambda\frac{S_2}{d_2}$$

$$a_{55} = -(\alpha(S_1 + S_2 + 2S_3) + \lambda\frac{S_2}{d_2})$$

$$\boldsymbol{B} = \frac{1}{mc} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T$$

Aluminum plate parameters are shown in the following.

Density of aluminum: $4500[kg/m^3]$
Specific heat of aluminum: $0.917[kJ/kgK]$
Heat transfer coefficient: $25[W/m^2K]$
Thermal conductivity: $238[W/mK]$
Wide of plate: $250[mm]$
Thick of plate: $10[mm]$
Longitudinal of plate: $120[mm]$
Peak watt of heater: $40[W]$

Then we can derive the following plant parameters as aluminum plate thermal process.

$$\begin{aligned} A[z^{-1}] &= 1 - 3.02z^{-1} + 3.452z^{-2} - 1.849z^{-3} \\ &\quad +0.4615z^{-4} - 0.04282z^{-5} \\ B[z^{-1}] &= 0.04862z^{-1} - 0.118z^{-2} + 0.1017z^{-3} \\ &\quad -0.03644z^{-4} + 0.004634z^{-5} \end{aligned}$$

5. EXAMPLE

The numerical simulation and the experiment result for the aluminum plate thermal process are compared. In the comparison, the feedback loop is cut after 500 [sec]. GPC parameters are set to be $N_1 = 1$, $N_2 = 2$, $Nu = 2$, $\lambda(j) = 0.02$. And reference signal is $w = 4$, which means the temperature increase from ambient temperature. From Fig. 2 to Fig. 5, it shows that GPC Eq. (10) is applied to the aluminum plate thermal model (Fig. 2 and Fig. 3) and its experimental device (Fig. 4 and Fig. 5). And from Fig. 6 to Fig. 9, these also give the simulation result (Fig. 6 and Fig. 7) and the experimental result (Fig. 8 and Fig. 9). From their results, it is found that the proposed method can give the same output as GPC Eq. (10) in the simulation and also track to the reference signal in the experimental result. From Fig. 10 to Fig. 13, it shows that GPC Eq. (10) is applied to the aluminum plate thermal model (Fig. 10 and Fig. 11) and the experimental device (Fig. 12 and Fig. 13) in the case that the feedback loop is cut after 500 [sec]. Although both simulation and experimental result show the derived system is stable, the output in the steady state becomes large value. So it is not desirable for safety. On the other hand, from Fig. 14 to Fig. 17, these also give the simulation result (Fig. 14 and Fig. 15) and the experimental result (Fig. 16 and Fig. 17) in the case that the feedback loop is cut after 500 [sec]. Both results show the proposed method can give stable system, and also the system output tries to keep remaining the closed-loop output. Therefore it can be seen that the proposed method is effective to design safety system.

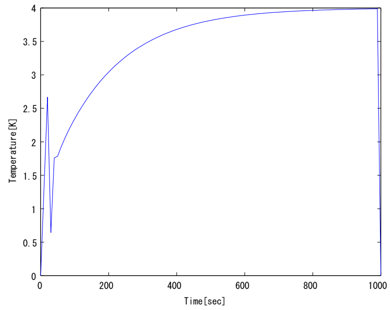

Fig. 2 Simulation result of output by GPC

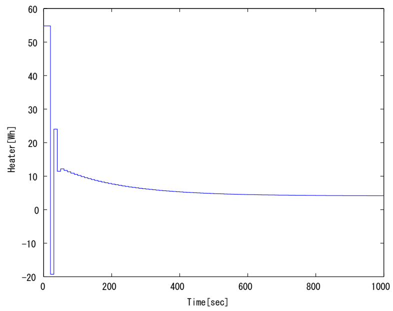

Fig. 3 Simulation result of input by GPC

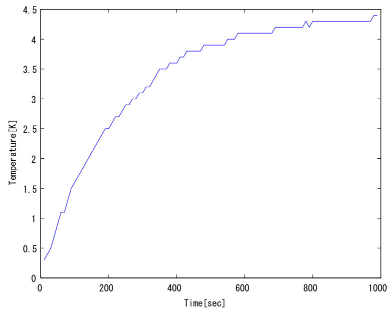

Fig. 4 Experimental result of output by GPC

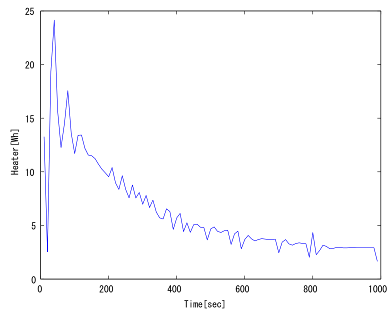

Fig. 5 Experimental result of input by GPC

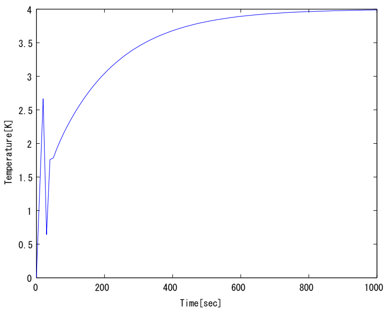

Fig. 6 Simulation result of output by proposed method

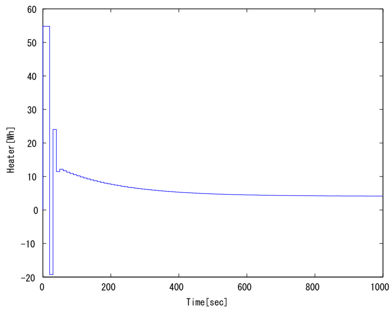

Fig. 7 Simulation result of input by proposed method

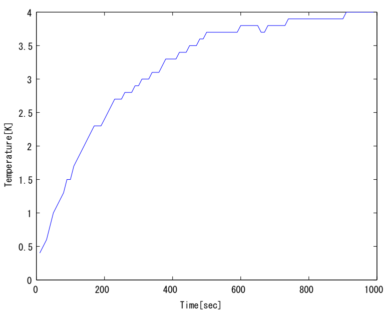

Fig. 8 Experimental result of output by proposed method

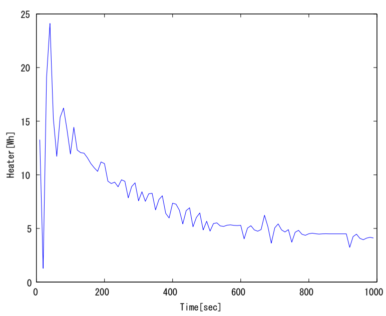

Fig. 9 Experimental result of input by proposed method

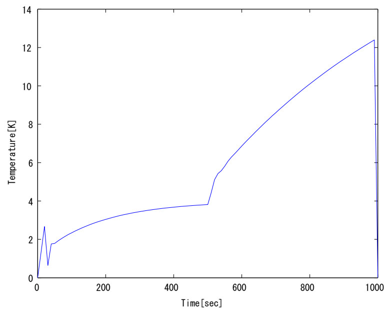

Fig. 10 Simulation result of output by GPC in cutting the feedback loop

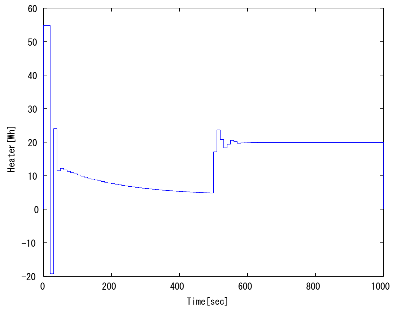

Fig. 11 Simulation result of input by GPC in cutting the feedback loop

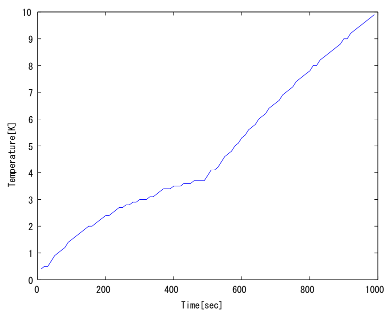

Fig. 12 Experimental result of output by GPC in cutting the feedback loop

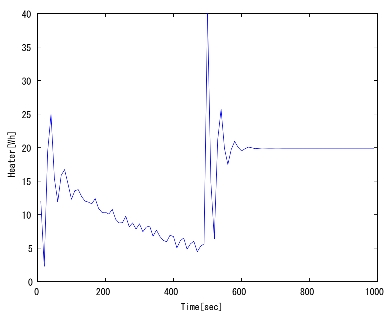

Fig. 13 Experimental result of input by GPC in cutting the feedback loop

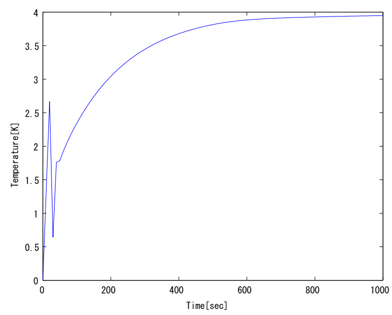

Fig. 14 Simulation result of output by proposed method in cutting the feedback loop

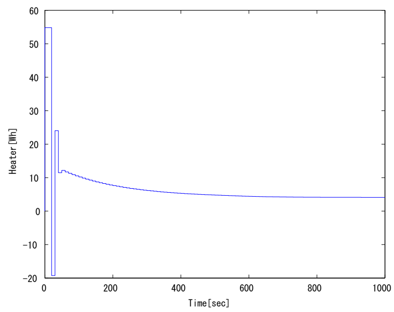

Fig. 15 Simulation result of input by proposed method in cutting the feedback loop

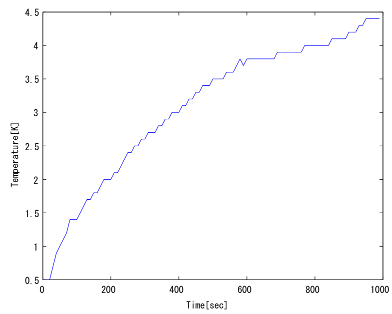

Fig. 16 Experimental result of output by proposed method in cutting the feedback loop

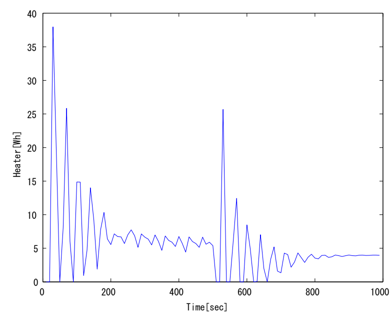

Fig. 17 Experimental result of input by proposed method in cutting the feedback loop

6. CONCLUSION

This paper explored an application of strongly stable generalized predictive control to temperature control of an aluminum plate, and a design method of strongly stable GPC focused on closed-loop characteristics is proposed. The proposed method has the feature that the steady state of output becomes the same as the steady state of closed-loop output even if feedback loop is cut. The effectiveness of the proposed method is shown by the numerical simulations and experimental results. The proposed method will be extended to multi-input multi-output systems or nonlinear systems as future works.

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